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Defining Coiled Tubing Limits - A New Approach

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Abstract

The burst, collapse and axial load operating limits for Coiled Tubing (CT) are currently established using the Von Mises incipient yield criterion. This criterion has historically been used to calculate the limits for oil country tubular goods (OCTG). The limits according to this criterion are based on the point at which the pipe material reaches a load state in which it begins to yield. Because of the bending that occurs when the CT is spooled on and off the reel, and when it is bent over the guide arch, the CT is already far beyond the yield point before it enters a well. Thus, this criterion does not really apply to CT.

This paper describes a research project currently in progress. The purpose of this project is to define a new set of CT limits based on criteria other than "incipient yield". This new approach to setting CT operating limits takes into account the internal residual stresses in the CT which are a consequence of repeated bending cycles and the accompanying change in material properties.

Introduction

The use of the Von Mises incipient yield criterion to establish CT limits is discussed in detail in Refs. 1 through 4. This criterion calculates the tri-axial stresses (axial, tangential and radial) caused by the forces applied on the CT due to the following:

- axial force (tensile or compressive),
- internal pressure,
- external pressure,
- bending due to helical buckling.

The tri-axial stresses caused by these externally applied forces and pressures are combined using the Von Mises criterion to calculate the Von Mises stress, σ_{vme}. This combined stress, σ_{vme}, is then compared to the yield stress, σ_{yp}, which is determined from a uni-axial pull test on a sample of the CT. When σ_{vme} reaches σ_{yp}, it is assumed that the CT material will begin to yield. This point of "incipient yield" is currently used by the CT industry to determine the burst, collapse, tensile and compressive limits that the CT can be subjected to.

However, this method ignores the internal residual stresses in the CT caused by the bending that occurs when the CT is bent on and off the reel and over the guide arch. These residual stresses cause yielding to begin much earlier than predicted by this method.

The material used to make CT has a well-defined yield point in the axial direction. However, beyond the yield point σ_{yp}, the *Bauschinger effect* and work softening change σ_{yp} and make it less well defined. Also, σ_{yp} in the tangential or hoop direction may not be the same as σ_{yp} in the axial direction.

The Gas Research Institute (GRI) has funded a research project to understand and model the material characteristics and to develop an improved method for setting CT limits. This paper describes the process that is being used in this project to accomplish these tasks. First, the residual stresses and curvature in CT due to the bending and straightening that occurs in CT strings are calculated. Experiments to determine the change of properties that occur with cyclic loading are then described, and results from preliminary tests are presented. Finally, the limitations of the incipient yield criterion and a proposed new method for developing CT limits are discussed.

Residual Stresses

When a length of coiled tubing is bent to a radius R_b and released, it returns to its initial straight position if $R_b \geq R_{b,y}$, or retains a residual radius of curvature* R_r, if $R_b < R_{b,y}$. Here, $R_{b,y}$, the yield radius is defined as[5]

$$R_{b,y} = (r_o E)/\sigma_{yp} \quad \text{.........(1)}$$

The following is an analysis of the residual stresses in CT that is bent, released, straightened, and released again. The analysis assumes that (i) the CT material is elastic perfectly plastic, (ii) plane sections remain plane during bending, straightening, and release, (iii) whenever the CT is released, the path of unloading of every fiber is elastic, (iv) the axis of the CT lies in the neutral plane, and the neutral surface does not shift at any stage, and (v) the bending radius is small enough for the elastic plastic boundary to have penetrated the inner radius of the CT+, i.e., $R_b \le r_i/\varepsilon_y$. Assumption (i) is not strictly valid (see Fig. 3 and section entitled "Material Properties). A more complete model must relax this assumption and account for the change in material properties. Assumption (iv) is questionable since the authors have found evidence which indicates shifting of the neutral surface during bending and subsequent straightening. However, the residual stresses calculated in this analysis indicate the order of magnitude of the expected stresses, and the method can be easily extended to account for the shift of the neutral surface.

Initial Bend

Upon completion of the initial bend, the strain profile across the cross section of the CT is given by

$$\varepsilon_b(y) = y/R_b \quad \text{.........(2)}$$

and the stress profile (for $R_b < R_{b,y}$) is given by

$$\sigma_b(y) = \begin{cases} Ey/R_b, & 0 \le y \le \varepsilon_y R_b, \\ \sigma_{yp}, & \varepsilon_y R_b \le y \le r_o, \end{cases} \quad \text{.........(3)}$$

$$\sigma_b(-y) = -\sigma_b(y)$$

If $R_b \le r_i/\varepsilon_y$, the bending moment is

$$M_b(R_b) = 0.5(E/R_b)\left[r_o^4 I_3(\varepsilon_y R_b, r_o) - r_i^4 I_3(\varepsilon_y R_b, r_i)\right] + (4/3)\sigma_{yp}\left[I_2(\varepsilon_y R_b, r_o) - I_2(\varepsilon_y R_b, r_i)\right] \quad \text{.......(4)}$$

where

$$I_2(t,a) = a^3\left[1-(t/a)^2\right]^{3/2}, \quad \text{.........(5)}$$

$$I_3(t,a) = \sin^{-1}(t/a) - 0.25\sin\{4\sin^{-1}(t/a)\} \quad \text{.........(6)}$$

* See Nomenclature for definitions.

+ Relaxing this assumption does not change the argument. The extension to cases where plasticity has not penetrated the inner radius is trivial. However, in problems that involve CT, assumption (v) holds true.

Equations 4, 5, and 6 agree with the result derived in Ref. 4, Eq. 14.

Release After Initial Bend

Releasing the CT is equivalent to applying a bending moment of $-M_b(R_b)$. The residual stress profile can therefore be calculated by superposing the stresses in the CT caused by the moments $M_b(R_b)$ and $-M_b(R_b)$. Since unloading is elastic (see Ref. 6, part I, p95 and part II, p377), the stress profile due to $-M_b(R_b)$ is given by

$$\sigma_{rel,b}(y) = -\left[M_b(R_b)/I_z\right]y \quad \text{.........(7)}$$

where I_z is the moment of inertia of the CT. The residual stress profile is

$$\sigma_{r,b}(y) = \sigma_b(y) + \sigma_{rel,b}(y). \quad \text{.........(8)}$$

The residual radius of curvature can be found by evaluating the strain in the outermost fibers that did not yield. These fibers are at distances of $\pm\varepsilon_y R_b$ (from the neutral axis), and the strain in them is always proportional to the curvature. Therefore, the residual radius is

$$R_{r,b} = \frac{\pm\varepsilon_y R_b}{\sigma_{r,b}(\pm\varepsilon_y R_b)/E}, \quad = \left[1/R_b - M_b(R_b)/(EI_z)\right]^{-1}. \quad \text{.........(9)}$$

It can be verified that Eq. (9) matches with Eq. (14) of Ref. 7.

Straightening

The characteristic feature of a beam that is bent beyond yield and subsequently straightened is a core section of unstressed fibers[8]. The stress profile in a CT that is straightened after being bent to a radius $R_b \le 0.5R_{b,y}$ # can be shown to be

$$\sigma_s(y) = \begin{cases} 0, & 0 \le y \le \varepsilon_y R_b, \\ -(Ey/R_b) + \sigma_{yp}, & \varepsilon_y R_b \le y \le 2\varepsilon_y R_b, \\ -\sigma_{yp}, & 2\varepsilon_y R_b \le y \le r_o, \end{cases} \quad \text{.........(10)}$$

$$\sigma_s(-y) = -\sigma_s(y).$$

The corresponding moment is

$$M_s(R_b) = M_b(R_b) - 0.5(E/R_b)\left[r_o^4 I_3(2\varepsilon_y R_b, r_o) + r_i^4 I_3(2\varepsilon_y R_b, r_i)\right] - (8/3)\sigma_{yp}\left[I_2(2\varepsilon_y R_b, r_o) - I_2(2\varepsilon_y R_b, r_i)\right], \quad \text{.........(11)}$$

where the functions I_2 and I_3 are defined in Eqs. (5) and (6) respectively and $M_b(R_b)$ is defined in Eq. (4).

In most CT situations the bending radius is less than 50% of the yield radius. Extension of the analysis to larger bending radii is trivial.

Releasing After Straightening

Releasing the CT from the straight position is equivalent to applying a moment equal to $-M_s(R_b)$. Since the path of unloading is elastic[6], the residual stress profile at equilibrium is the sum of the stress profile at the end of straightening and the stress profile $\sigma_{rel,s}(y)$ due to the moment $-M_s(R_b)$. This gives,

$$\sigma_{r,s}(y) = \sigma_s(y) - \left[M_s(R_b)/I_z\right]y \text{ ....(12)}$$

The residual curvature can be found from the strain in the outermost fibers that did not yield during loading or unloading. This radius of curvature can be shown to be

$$R_{r,s} = -EI_z/M_s(R_b) \text{ ....(13)}$$

As before, it can be verified that Eq. (13) is consistent with Eq. (14) of Ref. 7.

For 1.5" x 0.109" CT that has been bent to a radius of 48", released, straightened and released, the residual radius has a value of 20.2 ft. Figure 1 shows the stress profiles in this CT when it is (i) bent to 48" radius and held there, (ii) released, (iii) straightened, (iv) then released. The peak residual stresses for cases (ii) and (iv) are significant, in this case as much as 58% of the nominal yield stress. These residual stresses will influence the behavior of the CT for all subsequent loading.

Material Properties

If a metallic specimen is loaded beyond the yield point, subsequently unloaded, then loaded in the opposite direction, the material yield point in the opposite direction is reduced. This phenomenon is known as the *Bauschinger effect.* Since CT is routinely subjected to cyclic loading and unloading, the *Bauschinger effect* should be considered in the analysis of CT mechanics.

Figure 2 depicts an apparatus that was designed to hold a 3" test specimen of CT in an INSTRON universal testing machine. This machine applies cyclic axial strains on the specimen. The specimen has pressure ports through which internal pressure can be applied when the sample is subjected to strain controlled cyclic loading. An extensometer measures axial strain in the sample. This measurement is used to control the amount of strain applied on the specimen. Similar tests have been performed in the past by using "dog bone" specimens from the wall of the CT[9]. However, these tests did not allow the application of internal pressure.

A few initial tests were run with the apparatus shown in Fig. 2. Figure 3 shows the results from two of these tests conducted with zero internal pressure. In *Test A,* a 3" sample of 1.5" x 0.109" CT was (i) pulled (tensile loading) to seven times the yield strain[&], (ii) unloaded, compressed to zero strain, and (iii) pulled again to seven times the yield strain. Steps (i) to (iii) were repeated for 100 cycles. In *Test B,* the sample was first compressed to seven times the yield strain, unloaded, pulled to zero strain, and compressed to seven times the yield strain repeatedly. The test data in Fig. 3 indicates that the CT exhibits elastic perfectly plastic behavior only in its virgin state. Reversal of loading immediately changes the character of the stress-strain diagram. It is accompanied by strain softening and the *absence* of a sharply defined yield point.

The Bases for the Limit States of CT

A yield criterion describes the conditions under which yielding occurs in a body. Mathematically it can be represented by a function $f(\sigma_{i,j}, \sigma_{yp})$ where $\sigma_{i,j}$ defines the state of stress and σ_{yp} is the uni-axial yield strength in tension or compression[10]. When $f(\sigma_{i,j}, \sigma_{yp}) < 0$ the stress state is elastic. According to the incipient yield criterion, yielding begins when $f(\sigma_{i,j}, \sigma_{yp}) = 0$ at any point in the body. For a CT subjected to radial, hoop, and axial stresses, the Von Mises yield criterion states that

$$f = (\sigma_{rad} - \sigma_h)^2 + (\sigma_h - \sigma_a)^2 + (\sigma_a - \sigma_{rad})^2 - 2\sigma_{yp}^2 \text{ ....(14)}$$

The principal difficulty in applying such incipient yield criteria is that the limit state is determined by the first point that satisfies the condition $f(\sigma_{i,j}, \sigma_{yp}) < 0$. Secondly, the criterion assumes a sharply defined yield point.

Consider the case of a CT that has been bent and straightened at zero internal pressure ($\sigma_{rad} = \sigma_h = 0$). The stress profile across the cross section of the CT is shown in Fig. 1. All fibers at distances greater than $2\varepsilon_y R_b$ from either side of the neutral axis are at tensile or compressive yield stress. Hence, according to Eq. (14) the CT has already failed. However, in reality the CT continues to support axial loads. In fact, it can be shown that in the absence of internal pressure, the CT can support tensile axial loads of up to the yield load of the virgin CT[11, 12]. This can be explained by noting that the outer fibers which lie at distances less than $-2\varepsilon_y R_b$ from the neutral axis are at tensile yield stress and cannot support tensile loads. However, the rest of the fibers are capable of elastic deformation and can support axial loads. Thus, the incipient yield criterion has failed to predict (or has over-anticipated) the limit state of the CT. Despite significant residual stresses in (large) portions of the cross section, the CT still retains strength on a "global" basis. Since CT almost always retains significant residual stresses (see Fig. 1), the application of the incipient yield criterion will inevitably predict early onset of yield. In other words, "incipient yield" is too stringent a criterion. Clearly, the situation gets more complicated if the CT is subject to internal pressure as the

[&] This is approximately the strain in the outermost fibers from the neutral axis when 1.5" CT is bent to a radius of 48".

stress state is no longer uni-axial. Therefore, a criterion that determines a more "complete" and less stringent limit state for the CT must be determined.

When a body is subjected to stresses beyond the yield point, the state of stress due to further loading depends on (i) the initial state of stress, (ii) the path of loading (and hence the material property curve), and (iii) the geometry of the body. Furthermore, the order in which the body experiences different loading events becomes important. For example, bending followed by axial loading beyond the elastic limit is not the same as axial loading followed by bending beyond yield. Under these conditions, the failure/limit state of the CT must be determined by considering the factors mentioned above. Therefore, the procedure to determine the new CT limits should account for:

- residual stresses prior to a loading event,
- the change in material properties,
- "complete" failure (as opposed to local yielding) of the CT by including geometry dependent parameters.

The procedure should be based on rigorous analysis of CT mechanics and would have to be verified experimentally.

Test Apparatus

The test apparatus necessary to verify the CT limit states should be able to simulate the states of stress that are created in CT in the real world. It should be capable of fatiguing CT around different radii of curvature with or without internal pressure and also be capable of applying axial loads on fatigued CT. Furthermore, apparatus to study change in material properties due to cyclic loading of CT is required.

Figure 4 shows a Fatigue Test Machine (FTM) to simulate CT fatigue[13]. Test apparatus to measure change in material properties has been described earlier in this paper.

Figure 5 shows an Axial Loading Fixture (ALF) which operates in conjunction with a FTM by exerting an axial load on the CT when it is in a straight position. The ALF consists of a lower beam to which a support column is welded. The lower beam is attached rigidly to the FTM perpendicular to the plane of bending of the CT. An upper beam pivots on the support column via a pin assembly and can rotate in a plane normal to the plane of the figure. The CT test specimen is attached to the upper and lower beams of ALF at the right end. A load cell measures the load on the test specimen. A hydraulic ram attached at the left end is used to exert force on the upper beam. This force is amplified and transmitted as tensile load to the CT. Note that the CT must be in a "straight" position when it is attached to the ALF. A typical test procedure with this fixture consists of the following steps:

(i) Attach the lower beam of ALF to the fatigue machine.
(ii) Load the test specimen into the FTM (and pressurize it if necessary).
(iii) Fatigue the CT (i.e., bend and straighten) for the required number of cycles.
(iv) With the CT in the straight position, attach the upper beam of the ALF to the CT. Ensure that the FTM is holding the test specimen straight.
(v) Apply axial load by pressurizing the hydraulic cylinder.
(vi) Upon completion of loading, unload and detach upper beam from the CT and return control to the FTM.

The above test procedure is capable of creating a state of tri-axial stress in fatigued CT. This test and the material property tests should enable experimental verification of the new CT limits.

Design Methodolgy

Engineering design of a member is based on understanding the possible modes of its failure. According to traditional design philosophy, (known as Working Stress Design or WSD) the limits of design of a system are determined by a 'factor of safety', $SF = R_n / R_w$, where R_n is the nominal resistance and R_w is the safe working magnitude of a given parameter. R_n is determined from theory or experiment while R_w is chosen, based on experience and/or observation. To put it differently, this approach compares an "estimated" most severe loading condition that can occur on a system with its "known" least capacity. Safe design is assured by the appropriately chosen "safety factor" mentioned above.

In contrast, a reliability based design approach has its basis in probability. According to Payne and Swanson[14], in this approach, the expected load l on a system and its resistance (or capacity) c are both treated as random variables (as they are in reality). These random variables model the variability of design loads, material properties, and geometry of the structure. The variability (or uncertainty) in each factor is indicated by the statistical spread in the data. The goal then is to ensure that the capacity always exceeds the load. The design is said to have failed when $c < l$. The reliability of the design is quantified by the mathematical probability that c is always greater than l. In essence, the magnitude of $c - l\,(= g)$ defines the "limit state" of the system. The design is safe for positive values of g and unsafe for negative values. Therefore, the designer first selects a target probability of failure for the design. He also has available the probability distributions for the Load and Resistance factors of the structure. Based on this information, the designer chooses a set of design parameters and ensures that they satisfy a Design Check Equation (DCE). The choice of the load and resistance factors should be such that they result in the predetermined target probability of failure. References 14 - 17 contain further details of this method, which is known as Load and Resistance Factor Design (LRFD).

The authors believe that an LRFD based approach should be ultimately employed in defining CT limit states. The problem of determining the limit states of CT is essentially the inverse of a design problem. In a design problem a structure or member is designed to meet certain load requirements or reliability criteria. In this case, the limit state is a "known" quantity while the design variables are unknown. In the CT limit state problem, the limit states are the unknown quantities while the design variables and loads (or their probability distributions) are known. The probability of failure (or the limit state) must then be found. As pointed out earlier, the ultimate limit state of CT for a given load is a function of its initial or residual stress profile, its loading path, material properties and geometry. An LRFD based approach will eliminate the need for exact knowledge of these parameters during the determination of the limit states. Instead, the "risk of failure" for each loading condition can be assessed. This will require data gathering so that a statistically large enough sample is available to estimate the spread in the data.

To summarize, this project will try to determine CT limits by:

- first developing a more complete (as opposed to incipient) yield criterion based on rigorous analysis of the CT mechanics, and subsequent verification by experiments,
- incorporating these models in an LRFD based approach so that the risk of failure (or exceeding the limit state) of the CT for a given situation can be found.

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Nomenclature

c	Capacity or resistance of a structure or member, random variable
E	Young's modulus, psi
$I_2(t,a)$	Function defined in Eq. (5), in^3
$I_3(t,a)$	Function defined in Eq. (6)
I_z	Moment of inertia, $0.25\pi\left(r_o^4 - r_i^4\right)$, in^4
l	Expected load on a system, random variable
$M_b(R_b)$	Bending moment at radius R_b, ft-lbs
$M_s(R_b)$	Bending moment to hold CT straight, ft-lbs
r_o	Outer radius of CT, in
r_i	Inner radius of CT, in
R_b	Bending radius, in
$R_{b,y}$	Yield radius, in
R_n	Nominal resistance of a given parameter
R_w	Working resistance of a given parameter
$R_{r,b}$	Residual radius of curvature, bend and release, in
$R_{r,s}$	Residual radius of curvature after bend to straighten and release, in
SF	Safety factor
y	Distance of a fiber from the neutral axis, in
ε ; ε_y	strain; yield strain
σ	Stress, psi
σ_a	Axial stress, psi
σ_h	Hoop stress, psi
σ_{rad}	Radial stress, psi
$\sigma_{r,b}$	Residual stress after bend, release, psi
$\sigma_{r,s}$	Residual stress after bend, straighten, release, psi
σ_{vme}	Von Mises stress, psi
σ_{yp}	yield stress, psi

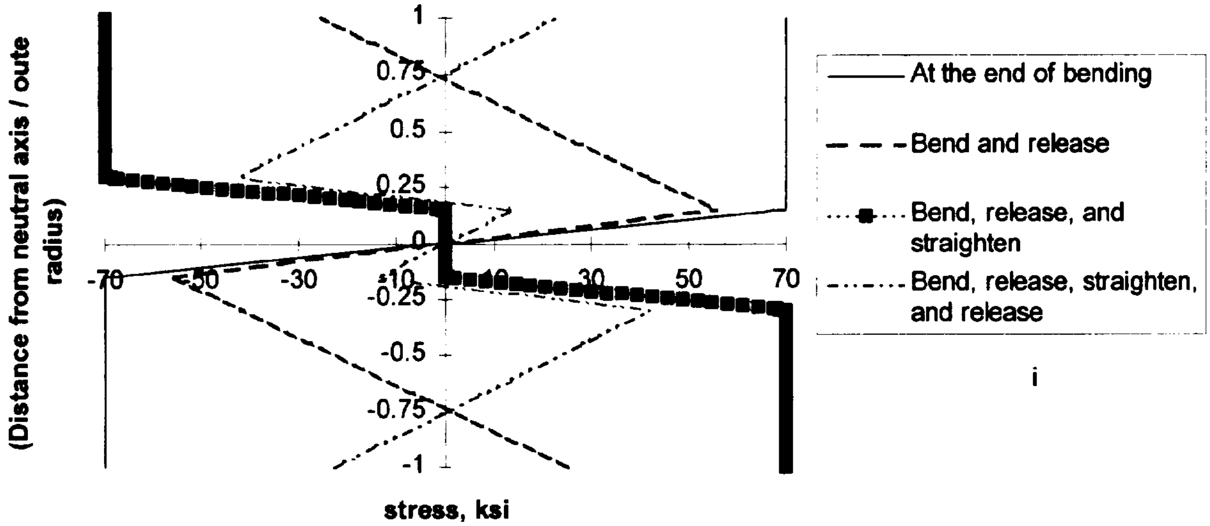

Figure 1 - Residual Stress
1.5" X 0.109" CT, 70 ksi, Bending Radius 48"

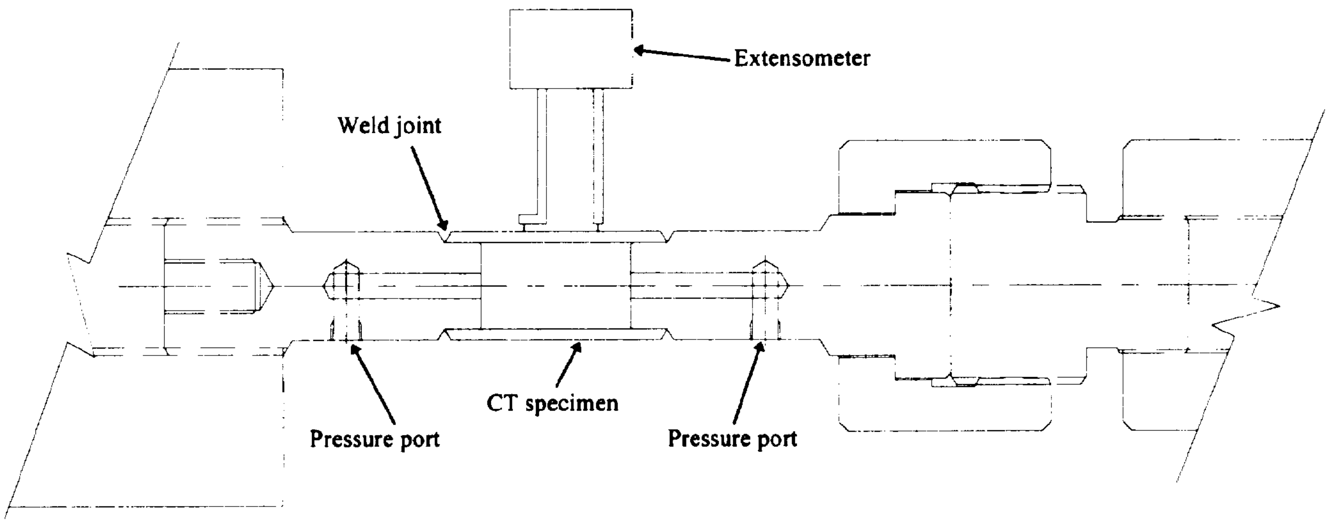

Figure 2 - Controlled Strain Test Specimen

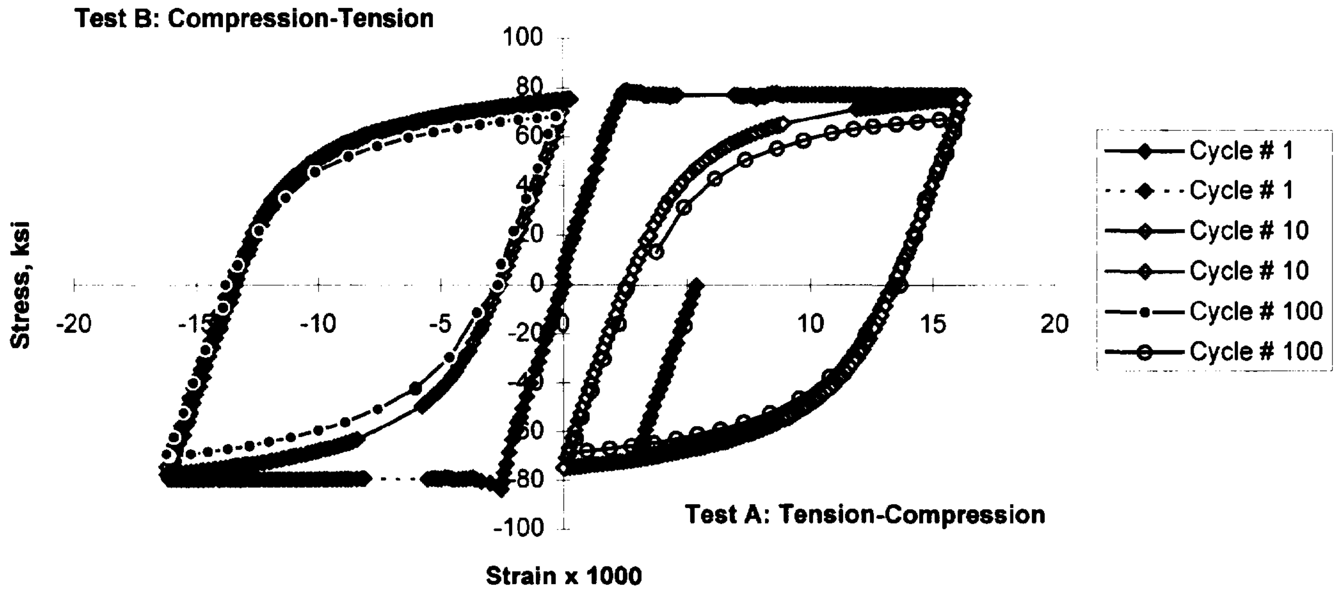

Figure 3 - Cyclic loading of 1.5" x 0.109" CT

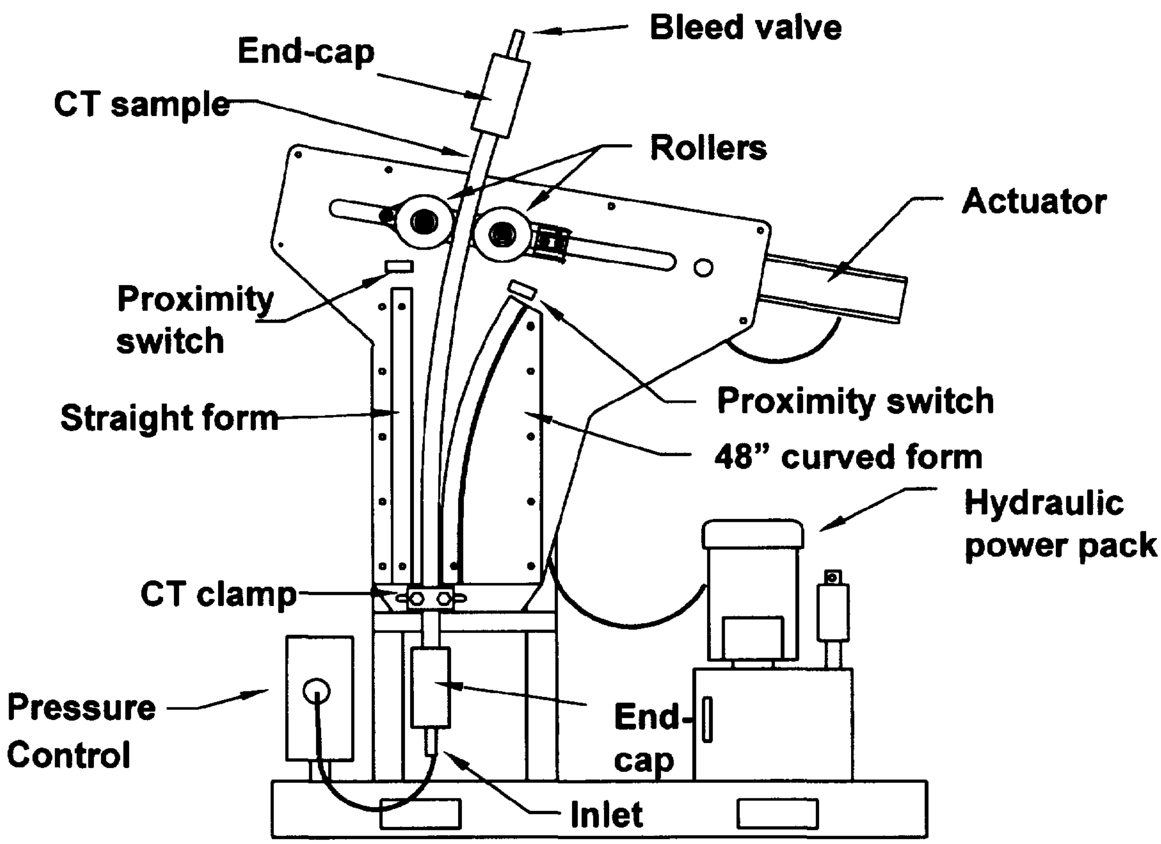

Figure 4 - Fatigue Test Machine (FTM)

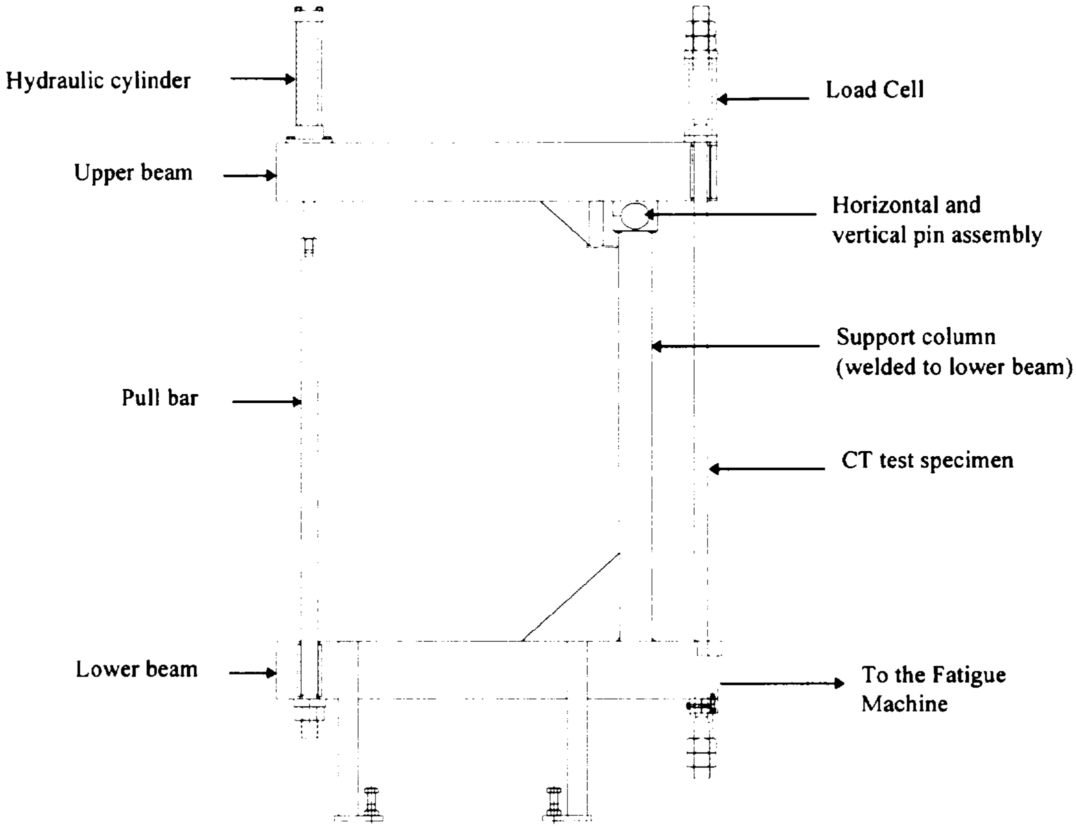

Figure 5 Axial Loading Fixture (ALF) for Fatigue Test Machine